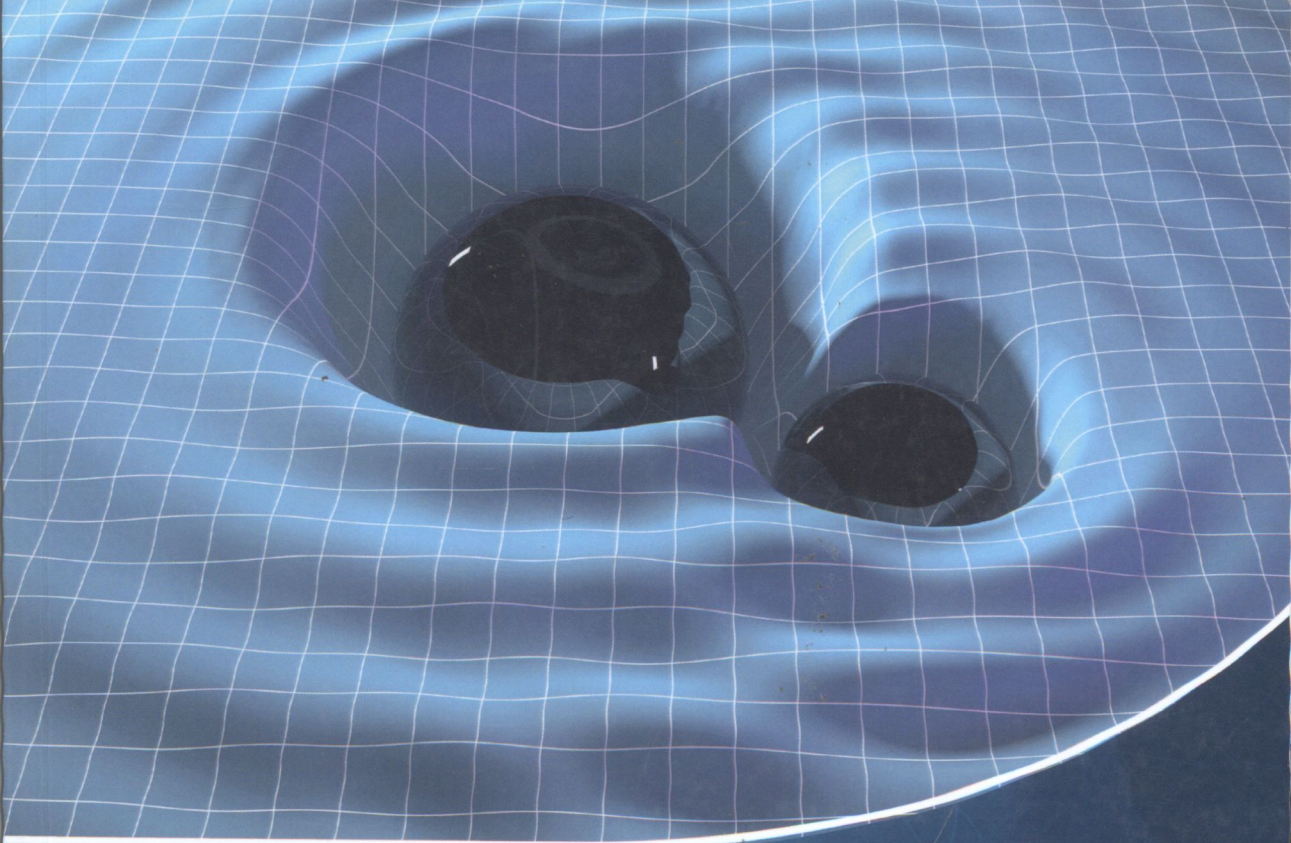




A General Relativity Coursebook

A GENERAL RELATIVITY COURSEBOOK

ED DAW
University of Sheffield



جامعة أمحمد بوقرة - بومرداس
Université M'hamed Bougara - Boumerdes
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'The approach in the book is unique, and especially valuable for the student first encountering general relativity. It shows in detail the computational steps involved in gaining the main results.'

Rainer Weiss, Nobel laureate; Professor Emeritus at MIT

'I think this is an excellent introduction to general relativity, and its important applications to cosmology and gravitational wave astrophysics, for the serious student who has not experienced the necessary mathematical formalism before and is willing to follow the text and attempt the many examples. It is an ideal lead-in to many of the more sophisticated modern textbooks which are now available.'

Sir James Hough OBE FRS FRSE; University of Glasgow

General relativity is a subject that most undergraduates in physics are particularly curious about, but it has a reputation for being very difficult. This book provides as gentle an introduction to general relativity as possible, leading you through the necessary mathematics in order to arrive at important results. Of course, you cannot avoid the mathematics of general relativity altogether, but, using this book, you can gain access to and appreciation of tensors and differential geometry at a pace you can keep up with. Early chapters build up to a complete derivation of Einstein's equations, while the final chapters cover the key applications on black holes, cosmology and gravitational waves. It is designed as a coursebook with just enough material to cover in a one-semester undergraduate class, but it is also accessible to any numerate readers who wish to appreciate the power and beauty of Einstein's creation for themselves.



Online Resources

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- Solutions manual for instructors
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Cover illustration: Mark Garlick/Science Photo Library/Getty Images.
An artist's conceptual illustration of two black holes orbiting each other, emitting gravitational waves, a prediction of Einstein's general theory of relativity.

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