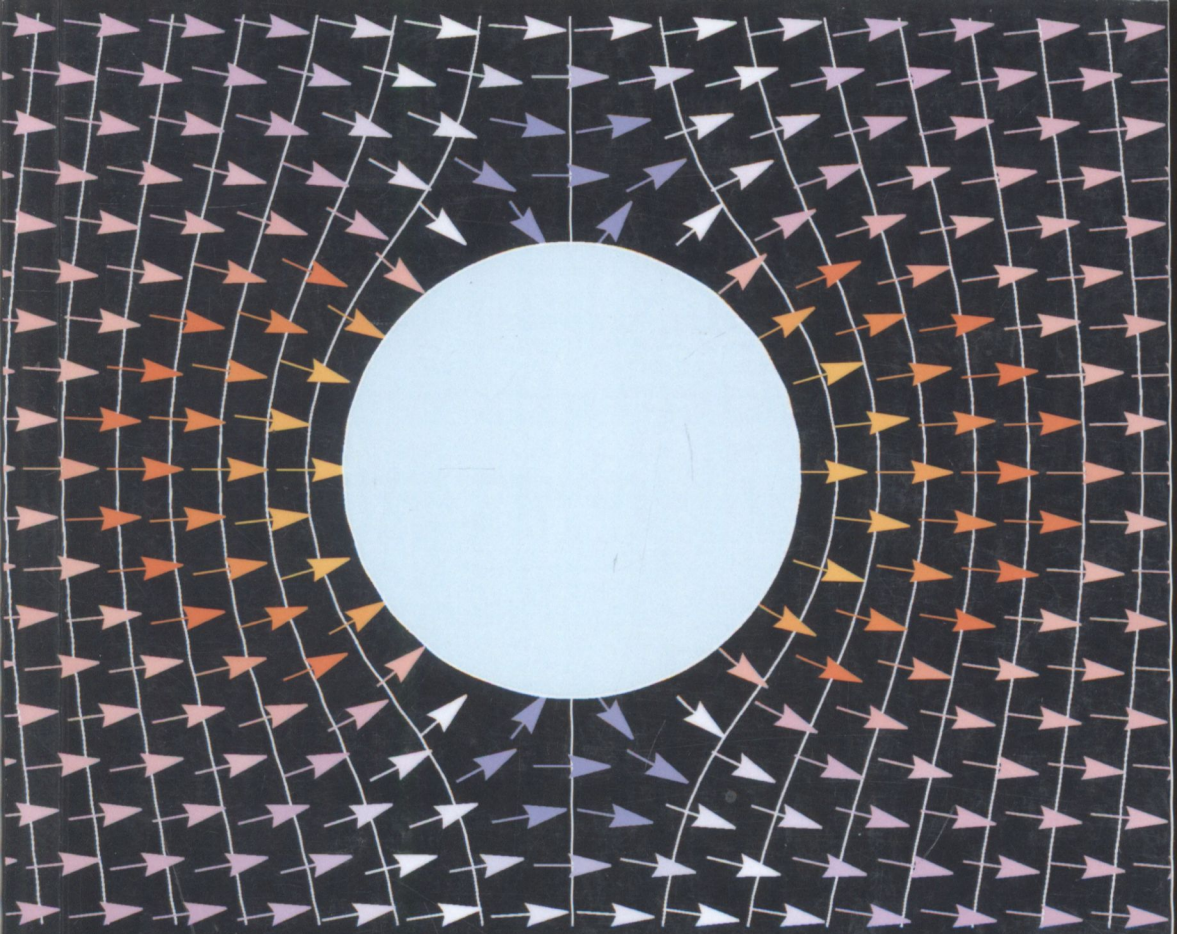




A Short Introduction to Mathematical Concepts in Physics

Jim Napolitano

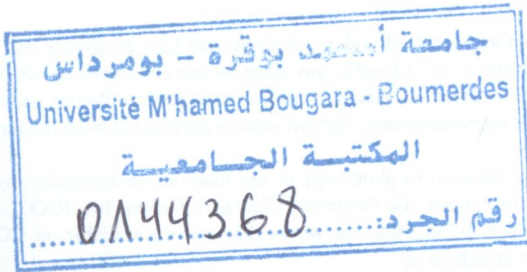


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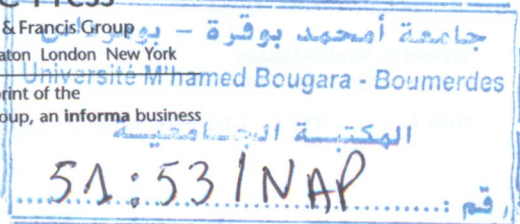
Jim Napolitano



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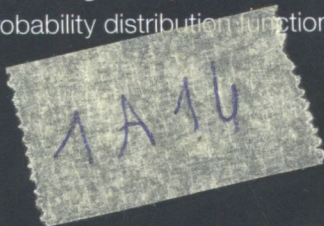
Mathematics is the language of physics and yet, mathematics is an enormous subject. This textbook provides an accessible and concise introduction to mathematical physics for undergraduate students taking a one semester course.

It assumes the reader has studied a year of introductory physics and three semesters of basic calculus, including some vector calculus, but no formal training in differential equations or matrix algebra. It equips readers with the skills and foundational knowledge they need for courses that follow in classical mechanics, electromagnetism, quantum mechanics, and thermal physics.

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