



A Student's Guide to  
**Special  
Relativity**

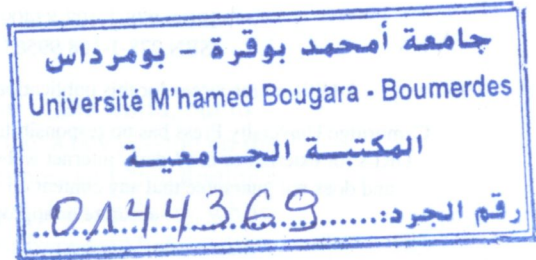
**NORMAN GRAY**



# A Student's Guide to Special Relativity

NORMAN GRAY

University of Glasgow



1eap

# Contents

---

|                                                               |         |
|---------------------------------------------------------------|---------|
| <i>Preface</i>                                                | page xi |
| <i>Acknowledgements</i>                                       | xv      |
| <i>Aims</i>                                                   | xvi     |
| <b>1 Introduction</b>                                         | 1       |
| 1.1 The Basic Ideas                                           | 1       |
| 1.2 Events                                                    | 3       |
| 1.3 Inertial Reference Frames                                 | 3       |
| 1.4 Simultaneity: Measuring Times                             | 7       |
| 1.5 Simultaneity: Measuring Lengths                           | 9       |
| 1.6 The Clock Hypothesis                                      | 10      |
| 1.7 Standard Configuration                                    | 12      |
| 1.8 Further Reading                                           | 13      |
| Exercises                                                     | 16      |
| <b>2 The Axioms</b>                                           | 18      |
| 2.1 The First Postulate: the Principle of Relativity          | 18      |
| 2.2 The Second Postulate: the Constancy of the Speed of Light | 27      |
| Exercises                                                     | 29      |
| <b>3 Length Contraction and Time Dilation</b>                 | 32      |
| 3.1 Simultaneity                                              | 32      |
| 3.2 Length Contraction and Time Dilation, Qualitatively       | 34      |
| 3.3 The Light Clock                                           | 37      |
| 3.4 The Horizontal Light Clock: Length Contraction            | 40      |
| 3.5 Is There Anything I Can Hold on To?                       | 42      |
| Exercises                                                     | 43      |
| <b>4 Spacetime and Geometry</b>                               | 45      |
| 4.1 Natural Units                                             | 46      |
| 4.2 The Minkowski Diagram                                     | 50      |



|          |                                                          |     |
|----------|----------------------------------------------------------|-----|
| 4.3      | Plane Rotations                                          | 55  |
| 4.4      | The Invariant Interval                                   | 56  |
| 4.5      | Changes of Frame, and Perspective                        | 61  |
| 4.6      | Length and Time in the Minkowski Diagram                 | 63  |
| 4.7      | Worldlines and Causality                                 | 66  |
|          | Exercises                                                | 68  |
| <b>5</b> | <b>The Lorentz Transformation</b>                        | 72  |
| 5.1      | The Derivation of the Lorentz Transformation             | 72  |
| 5.2      | Addition of Velocities                                   | 75  |
| 5.3      | The Invariant Interval and the Geometry of Spacetime     | 77  |
| 5.4      | Proper Time and the Invariant Interval                   | 78  |
| 5.5      | Applications of the Lorentz Transformation               | 79  |
| 5.6      | The Equations of Special Relativity in Physical Units    | 83  |
| 5.7      | Paradoxes                                                | 84  |
| 5.8      | Some Comments on the Lorentz Transformation              | 93  |
|          | Exercises                                                | 98  |
| <b>6</b> | <b>Vectors and Kinematics</b>                            | 103 |
| 6.1      | Three-Vectors                                            | 103 |
| 6.2      | Four-Vectors                                             | 105 |
| 6.3      | Velocity and Acceleration                                | 110 |
| 6.4      | Velocities and Tangent Vectors                           | 114 |
| 6.5      | The Frequency Vector, and the Doppler Shift              | 115 |
|          | Exercises                                                | 118 |
| <b>7</b> | <b>Dynamics</b>                                          | 125 |
| 7.1      | Energy and Momentum                                      | 125 |
| 7.2      | Photons                                                  | 131 |
| 7.3      | Relativistic Collisions and the Centre-of-Momentum Frame | 132 |
| 7.4      | But Where's This Mass Coming From?                       | 136 |
| 7.5      | More Unit Fun: an Aside on Electron-volts                | 137 |
| 7.6      | Relativistic Force                                       | 138 |
| 7.7      | An Example: Compton Scattering                           | 139 |
| 7.8      | Not The End                                              | 140 |
|          | Exercises                                                | 141 |
|          | <b>Appendix A An Overview of General Relativity</b>      | 145 |
| A.1      | Some Thought Experiments on Gravitation                  | 146 |
| A.2      | Geometry                                                 | 153 |
| A.3      | Gravity                                                  | 160 |
| A.4      | Solutions of Einstein's Equation                         | 163 |

|                   |                                                        |     |
|-------------------|--------------------------------------------------------|-----|
| <i>Appendix B</i> | Relativity's Contact with Experimental Fact            | 177 |
| B.1               | Special Relativity                                     | 178 |
| B.2               | General Relativity: Classical and Post-classical Tests | 183 |
| B.3               | A Closer Look at the 1919 Eclipse Observations         | 186 |
| <i>Appendix C</i> | Maths Revision                                         | 194 |
| C.1               | Complex Numbers                                        | 194 |
| C.2               | The Hyperbolic Functions                               | 196 |
| C.3               | Linear Algebra                                         | 198 |
| <i>Appendix D</i> | How to Do Calculations: a Recipe                       | 200 |
| D.1               | Key Things to Remember                                 | 200 |
| D.2               | A Checklist for Relativity Problems                    | 200 |
| D.3               | Which Equation When?                                   | 202 |
| D.4               | Rest, Moving and Stationary Frames: Be Careful!        | 203 |
| D.5               | Length Contraction, Time Dilation, and 'Rest Frames'   | 204 |
|                   | <i>References</i>                                      | 205 |
|                   | <i>Index</i>                                           | 211 |

This compact yet informative Guide presents an accessible route through Special Relativity, taking a modern axiomatic and geometrical approach. It begins by explaining key concepts and introducing Einstein's postulates. The consequences of the postulates – length contraction and time dilation – are unravelled qualitatively and then quantitatively. These strands are then tied together using the mathematical framework of the Lorentz transformation, before applying these ideas to kinematics and dynamics. This volume demonstrates the essential simplicity of the core ideas of Special Relativity, while acknowledging the challenges of developing new intuitions and dealing with the apparent paradoxes that arise. A valuable supplementary resource for intermediate undergraduates, as well as independent learners with some technical background, the Guide includes numerous exercises with hints and notes provided online. It lays the foundations for further study in General Relativity, which is introduced briefly in an appendix.

**Norman Gray** is in the School of Physics and Astronomy, University of Glasgow, where he has, over the last 20 years, taught courses covering both special and general relativity. Since his PhD in particle theory, he has had a varied research career, most recently concentrating on the interface between astronomy and computing. He is author of *A Student's Guide to General Relativity* (2019), which has been a successful addition to Cambridge University Press's growing series of Student's Guides.



## Online Resources

**[www.cambridge.org/gray-sgsr](http://www.cambridge.org/gray-sgsr)**

- Problems with solutions
- For more information and other books in the series:  
[cambridge.org/ASGT](http://cambridge.org/ASGT)

Image: John Abbate/Moment Open/Getty Images.  
Cover design by Zoe Naylor.

**CAMBRIDGE**  
UNIVERSITY PRESS  
[www.cambridge.org](http://www.cambridge.org)

ISBN 978-1-108-99563-4



9 781108 995634 >

