

Classical and Quantum Statistical Physics

Fundamentals and Advanced Topics

Carlo Heissenberg
and Augusto Sagnotti





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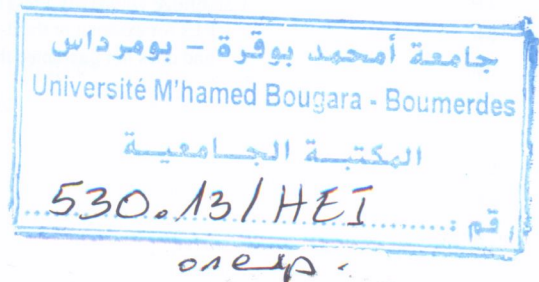
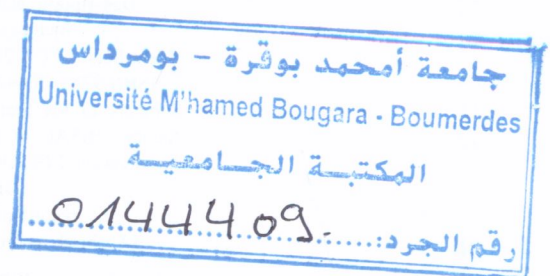
Fundamentals and Advanced Topics

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"The concepts and methods of statistical physics play a key role, not always fully perceived, in all branches of physics. With this textbook, aimed primarily at advanced undergraduates but also useful for experienced researchers, Heissenberg and Sagnotti explain clearly and convincingly why it is so. Besides presenting a modern exposition of the basic facts of statistical physics well equipped with problems, a stimulating and broad range of advanced topics is introduced, whetting the appetite of the determined reader and pushing them to go farther to quantum field theory and mathematical physics."

Prof. Roberto Raimondi, Università Roma Tre

"In its presentation of statistical mechanics, this book is unique for its emphasis on the quantum mechanical underpinnings. It would make a great text for a multidisciplinary course on many-body physics for upper-division undergraduates or beginning graduate students. Even in the more-elementary first half, the book is full of underappreciated gems, and gives glimpses of a broad view of theoretical physics as a whole. The second half boasts a uniform and elementary treatment of the Onsager solution, the Bethe ansatz, the Renormalization Group, and the approach to equilibrium."

Prof. John McGreevy, University of California San Diego

Statistical physics examines the collective properties of large ensembles of particles, and is a powerful theoretical tool with important applications across many different scientific disciplines. This book provides a detailed introduction to classical and quantum statistical physics, including links to topics at the frontiers of current research. The first part of the book introduces classical ensembles, provides an extensive review of quantum mechanics, and explains how their combination leads directly to the theory of Bose and Fermi gases. This allows a detailed analysis of the quantum properties of matter and introduces the exotic features of vacuum fluctuations. The second part discusses more advanced topics such as the two-dimensional Ising model and quantum spin chains. This modern text is ideal for advanced undergraduate and graduate students interested in the role of statistical physics in current research. One hundred and forty homework problems reinforce key concepts and further develop readers' understanding of the subject.

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